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# DIVISION OF LABOUR IN THE TECHNOLOGY OF EXCHANGE

by

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## 1 Introduction.

**A** The traditional neo-classical models of an economy assume that no resources are used up when goods are exchanged. An individual is distinguished by his preferences and by his initial endowment of goods. He exchanges goods with respect to a parametric vector of prices to obtain the most preferred bundle of goods within his budget set.

Following Foley [6], Hahn [7], and Kurz [11] let us suppose that resources are consumed when goods are exchanged. Suppose here, also, that each individual possesses a certain degree of efficiency at exchanging goods with others. For example, individuals might have different abilities at searching for potential trading partners, at bargaining, or at transporting goods to and from the location where goods are physically transferred from the possession of one individual to another.

I will represent an individual's ability at executing exchange by his transaction technology. It describes all his feasible exchanges and their attendant resource costs. Under these assumptions, I will describe two ways of organizing exchange activities.

**B** The first method does not permit specialization or division of labour in the technology of exchange. Each individual is required to execute his own exchanges and to bear directly any transaction costs incurred. His desired exchanges will be based on his preferences, his existing stock of goods, his beliefs about the prevailing exchange ratios between goods, and his transaction technology.

The second method permits specialization. An individual is permitted to execute exchanges on behalf of others. He may act as a trader by buying goods from some individuals and reselling them to others. By acting as a trader, an individual hopes to consume a more desirable bundle of goods than he could have if he only exchanged goods on his own behalf, or, if he permitted some trader to execute his exchanges. Competition between traders will ensure that only the relatively efficient individuals act as traders.

**C** Two equilibrium concepts, the core and the set of competitive price allocations, are considered for each method of exchanging goods. The core of an economy is the set of outcomes that no coalition of agents can improve upon. A core allocation is a distribution of goods which is acceptable to every individual and to every coalition or group of individuals. Its definition is “institutionally free” in the sense that it does not depend on the existence of a parametric list of prices.

It is well known that if exchange is costless each competitive equilibrium allocation, the distribution of goods among individuals after exchanging goods at equilibrium prices, belongs to the core. For economies with a finite number of individuals, the core is generally larger than the set of competitive allocations.

Debreu and Scarf [5] have shown that as the number of individuals “gets “large,” the core “shrinks” to the set of competitive allocations. The “equivalence” between the core and the set of competitive allocations has been demonstrated by Aumann [1] for a pure exchange economy and Hildenbrand [9] for a coalition production economy. The last two results depend on the assumption that an individual is insignificant in the sense that his presence does not significantly affect the outcome of exchanging goods.

**D** In this paper, I will deduce what the structure of prices must be, for each method of exchanging goods, such that a competitive equilibrium allocation is also a core allocation.

When division of labour is not permitted, an individual’s budget set depends on his current endowment of goods, on his transaction technology and on the ratios at which goods are currently exchanged. If individuals have different transaction technologies, they will also have different effective exchange ratios, even though the prevailing exchange ratios are consistent, and can be reduced to one price per good.

With traders, on the other hand, both buying and selling prices are required for each good. The differential between these prices reflects the transaction costs borne by the traders.

Competition between traders will ensure that a consistent set of buying and selling prices will prevail. An individual's budget set depends on these prices, and on his profit as a trader with respect to these prices. An individual will only operate as a trader if his profit is non-negative.

E I will use the theory of the core for two other purposes. First, I will show that a coalition, either with or without division of labour in its technology of exchange, can organize its members as effectively with a price system as it can by assigning exchange tasks to them.

In fact, profit maximization on the part of a coalition with division of labour, is equivalent to profit maximization on the part of individual traders.

Second, I will model the selection of a coalition's exchange technology using the theory of the core. I will look at the circumstances that lead to the selection of exchange with division of labour over exchange without division of labour.

F Notation.

- $A$  = the set of all individuals (agents) in the economy.
- $\Omega$  = the set of feasible coalitions, subsets of  $A$ , including  $A$ .
- $R_+^I$  = the non-negative orthant of the Euclidean space of dimension, " $I$ " – the commodity space.
- $I$  = the number of commodities or goods.
- $\underline{w}(a)$  = the initial endowment of individual  $a \in A$ , a vector in  $R_+^I$ , i.e.  $\underline{w}(a) \in R_+^I$ .
- $X(a)$  = the consumption set for individual,  $a \in A$ , a convex subset of  $R_+^I$ .
- $\underline{c}(a)$  = the final consumption bundle of goods for individual,  $a \in A$ , a vector in  $X(a)$ .
- $\cong_a$  = the preference ordering for individual,  $a$ , defined on  $X(a)$ . This ordering is transitive, reflexive, continuous, and complete. From  $\cong_a$  define the ordering  $\approx_a$  by  $\underline{s} \approx_a \underline{t}$  if  $\underline{s} \cong_a \underline{t}$  but not  $\underline{t} \cong_a \underline{s}$ , for vectors of goods,  $\underline{s}, \underline{t} \in X(a)$ .

## 2 Exchange Without Division of Labour.

A Transaction Sets and Exchange.

Express individual  $a$ 's efficiency at exchanging goods by his transaction set,  $B(a)$ , which I postulate to be a subset of  $R_+^{3I}$ . The exchange of the vector,  $\underline{y}(a) \in R_+^I$ , for the vector,  $\underline{x}(a) \in R_+^I$ , is said to be technologically feasible for individual,  $a$ , if a vector of goods,  $\underline{z}(a) \in R_+^I$ , exists such that:

$$(2.1) \quad (\underline{x}(a), \underline{y}(a), \underline{z}(a)) \in B(a).$$

Vector  $\underline{z}(a)$  represents the quantities of goods needed by agent,  $a$ , to complete the transaction. If he is successful, then his resulting consumption bundles,  $\underline{c}(a)$ , is given by:

$$(2.2) \quad \underline{c}(a) = \underline{w}(a) + \underline{x}(a) - \underline{y}(a) - \underline{z}(a).$$

Relations (2.1) and (2.2) express the constraints placed on exchanges of goods in this economy. An agent's exchanges are constrained both by his transaction set and by goods currently in his possession.

I assume the following properties hold simultaneously for the transaction set,  $B(a)$ , for every agent  $a \in A$ .

(B.1) The zero vector,  $\underline{0} \in B(a)$ ,

(B.2) If  $(\underline{x}(a), \underline{y}(a), \underline{z}(a)) \in B(a)$  then  $\underline{x}'(a) \leq \underline{x}(a)$ ,  $\underline{z}'(a) \geq \underline{z}(a)$  implies that  $(\underline{x}'(a), \underline{y}(a), \underline{z}'(a)) \in B(a)$ ,

(B.3)  $B(a)$  is a closed, bounded set,

(B.4)  $B(a)$  is a convex set.

Property (B.1) admits no exchange; property (B.2) admits free disposal of goods. Property (B.3) ensures that an agent's scale of operations is finite; property (B.4) implies non-increasing marginal returns.

**Definition 2.1 Allocation of Goods.**

An allocation of goods, denoted by  $c$ , is a distribution of goods among the individuals in the economy where  $\underline{c}(a)$  the vector assigned to individual  $a$ , is an element of  $X(a)$ .

**Definition 2.2 Attainable Allocation.**

An allocation,  $c$ , is said to be attainable by a coalition,  $E \in \Omega$  if for each agent  $a \in E$ , vectors  $\underline{x}(a)$ ,  $\underline{y}(a)$ , and  $\underline{z}(a)$  exist in  $R_+^I$ , such (2.1) and (2.2) are satisfied, and:

$$(2.3) \quad \sum_{(a \in E)} \underline{c}(a) = \sum_{(a \in E)} \underline{w}(a) - \sum_{(a \in E)} \underline{z}(a).$$

The last equation is coalition  $E$ 's material balance equation. Notice that (2.2) and (2.3) imply:

$$(2.4) \quad \sum_{(a \in E)} \underline{x}(a) = \sum_{(a \in E)} \underline{y}(a).$$

In other words, the total quantity of each good exchanged within a coalition must be mutually consistent.

An allocation that is attainable by the coalition  $A$ , the set of all agents, is called a state of the economy.

## B The Core.

The core, or the set of core allocations, is based on the following rationality postulate. Let  $c$  be an allocation which is attainable by the coalition of all agents of the economy,  $A$ . Then,  $c$  is said to be blocked by a coalition,  $E$ , if an allocation,  $h$ , exists attainable by  $E$  such that:

$$c(a) \cong_a h(a) \text{ for all } a \in E, \text{ and } c(a') \approx_{a'} h(a') \text{ for some agent } a' \in E.$$

Definition 2.3 Core Allocation.

An allocation,  $c$ , which is attainable by  $A$  and which cannot be blocked by any coalition,  $E \in \Omega$ , is said to be a core allocation.

The core is the set of all core allocations.

## C The Price System.

Consider a pure exchange economy where exchange does not consume real resources. With " $l$ " goods in the economy, there will be  $l(l-1)/2$  exchange ratios between goods. If arbitrage is effective or assumed, it is possible to reduce these exchange ratios to a set of  $l-1$  relative prices, denoted by the vector  $\underline{p} \in R_+^l$ , where any good can be the numeraire.

If exchange consumes real resources, things are more complicated. I have mentioned that the effective exchange ratios faced by an individual in this economy depend not only on the ratios at which quantities are transferred from the possession of one individual to another, but also on the individual's transaction set. If individuals have different transaction sets, their effective exchange ratios will differ as well.

Let the unit price simplex,  $\Delta \subseteq R_+^l$  be defined by:

$$\Delta = \{ \underline{p} \in R_+^l \mid \sum_{i=1, \dots, l} p^i = 1 \},$$

where the vector  $\underline{p}$ , represents a set of  $l-1$  relative prices.

Definition 2.4 Budget Set.<sup>1</sup>

Individual  $a$ 's budget set,  $\beta(a, \underline{p})$ , with respect to a price vector  $\underline{p} \in \Delta$  is given by:

$$\begin{aligned} & \mid a) \quad (\underline{x}(a), \underline{y}(a), \underline{z}(a)) \in B(a) \quad \quad \quad \}, \\ (2.6) \quad \beta(a, \underline{p}) = & \{ (\underline{x}(a), \underline{y}(a), \underline{z}(a)) \mid b) \quad \underline{w}(a) + \underline{x}(a) - \underline{y}(a) - \underline{z}(a) \in X(a) \}, \\ & \mid c) \quad \underline{p} \cdot \underline{x}(a) \leq \underline{p} \cdot \underline{y}(a) \quad \quad \quad \}. \end{aligned}$$

Part c) of the definition means that the value received is less than or equal to the value given up in exchange.

Definition 2.5 Demand Set.

Individual  $a$ 's demand set,  $\delta(a, \underline{p})$ , with respect to  $\underline{p} \in \Delta$  is given by:

$$(2.7) \quad \delta(a, \underline{p}) = \{ (\underline{x}(a), \underline{y}(a), \underline{z}(a)) \in \beta(a) \mid \text{for every } (\underline{x}'(a), \underline{y}'(a), \underline{z}'(a)) \in \beta(a), \\ \text{we have } \underline{w}(a) + \underline{x}'(a) - \underline{y}'(a) - \underline{z}'(a) \cong_a \underline{w}(a) + \underline{x}(a) - \underline{y}(a) - \underline{z}(a) \}.$$

Definition 2.6 Competitive Price Equilibrium.

The pair  $(\underline{p}, c)$  forms a competitive price equilibrium where  $\underline{p} \in \Delta$ , if vectors  $\underline{x}(a), \underline{y}(a), \underline{z}(a)$  exist for all  $a \in A$ , such that:

$$\underline{c}(a) = \underline{w}(a) + \underline{x}(a) - \underline{y}(a) - \underline{z}(a),$$

$$(2.8) \quad \underline{x}(a), \underline{y}(a), \underline{z}(a) \in \delta(a, \underline{p}), \text{ and}$$

$$(2.9) \quad \sum_{(a \in A)} \underline{c}(a) = \sum_{(a \in A)} \underline{w}(a) - \sum_{(a \in A)} \underline{z}(a).$$

The allocation,  $c$ , is called a competitive price equilibrium allocation.

See Wiens [18] for the conditions under which a competitive price equilibrium exists.

The straightforward (simple) proof of the following theorem is based on Hildenbrand [9].

Theorem 2.1.

Every competitive price equilibrium is also a core allocation.

Proof.

Let  $c$  be a competitive equilibrium allocation with respect to a price vector  $\underline{p} \in \Delta$ . Suppose it is not a core allocation.

Then, an allocation,  $h$ , exists attainable by some coalition,  $E$ , blocking  $c$ .

Formally,  $\underline{x}'(a), \underline{y}'(a), \underline{z}'(a) \in R_+^I$  exist such that:

$$(2.10) \quad \underline{h}(a) = \underline{w}(a) + \underline{x}'(a) - \underline{y}'(a) - \underline{z}'(a) \text{ all } a \in E,$$

$$(2.11) \quad (\underline{x}'(a), \underline{y}'(a), \underline{z}'(a)) \in B(a) \text{ all } a \in E,$$

$$(2.12) \quad \sum_{(a \in E)} \underline{h}(a) = \sum_{(a \in E)} \underline{w}(a) - \sum_{(a \in E)} \underline{z}'(a),$$

$$(2.13) \quad \underline{c}(a) \cong_a \underline{h}(a) \text{ all } a \in E,$$

$$(2.14) \quad \underline{c}(a') \approx_{a'} \underline{h}(a') \text{ for some } a' \in A.$$

But  $\underline{c}(a) \in \delta(a, \underline{p})$  for all  $a \in A$  plus (2.13) and (2.14) imply:

$$\underline{p} \cdot \underline{x}'(a) \geq \underline{p} \cdot \underline{y}'(a) \text{ all } a \in E, \text{ and}$$

$$\underline{p} \cdot \underline{x}'(a') > \underline{p} \cdot \underline{y}'(a').$$

The last two inequalities imply:

$$\sum_{(a \in E)} \underline{x}'(a) > \sum_{(a \in E)} \underline{y}'(a),$$

contradicting (2.10) and 2.12).

Q.E.D. Theorem 2.1.

Note that for economies where  $A$  is finite, core allocations may exist which cannot be achieved by exchanging goods with respect to a set of  $l-1$  relative prices.<sup>2</sup>

**E Aggregate Exchange Technologies.**

Because each individual executes his own exchanges, the aggregate exchange technology for a coalition,  $E$ , has the dimension of three times the cardinality of  $E$ . It is given by  $\{X_{(a \in E)} B(a)\}$ , where  $X$  indicates Cartesian (Cross) product.

Below, I want to compare aggregate exchange technologies without division of labour with those with division of labour. For this reason, I represent the former in the space  $R_+^{2l}$ .

For all  $E \in \Omega$ , define the set  $AB(E)$  by:

$$AB(E) = \{(\underline{u}, \underline{v}) \mid \underline{u}, \underline{v} \in R_+^l \text{ and such that an allocation, } c, \text{ exists attainable by } E, \\ \text{with } \underline{u} = \sum_{(a \in E)} [\underline{c}(a) - \underline{w}(a)]^+, \underline{v} = \sum_{(a \in E)} [\underline{c}(a) - \underline{w}(a)]^-\},$$

where:  $[\underline{c}(a) - \underline{w}(a)]^+ = \{(\underline{c}(a) - \underline{w}(a))_i \text{ if } (\underline{c}(a) - \underline{w}(a))_i > 0; 0 \text{ otherwise}\}$ .

$$[\underline{c}(a) - \underline{w}(a)]^- = \{-(\underline{c}(a) - \underline{w}(a))_i \text{ if } (\underline{c}(a) - \underline{w}(a))_i < 0; 0 \text{ otherwise}\}.$$

When agent,  $a$ , exchanges the vector  $\underline{y}(a)$  for  $\underline{x}(a)$ , the vector,  $\underline{z}(a)$ , of goods are used up. The vector,  $\underline{z}(a)$  consists of goods from  $a$ 's initial endowment and goods obtained from others during the process of exchange. Denote these quantities by the vectors,  $\underline{z}_1(a)$  and  $\underline{z}_2(a)$ , respectively, where  $\underline{z}(a) = \underline{z}_1(a) + \underline{z}_2(a)$ .

In effect, agent,  $a$ , gives up the vector  $\underline{y}(a) + \underline{z}_1(a)$  to obtain  $\underline{x}(a) - \underline{z}_2(a)$  for consumption.

If agent,  $a$ , is constrained to using only goods from his initial endowment to realize the exchange of  $\underline{y}(a)$  for  $\underline{x}(a)$ , then  $\underline{z}_2(a) = 0$ .

With these definitions:

$$[\underline{c}(a) - \underline{w}(a)]^+ = \underline{x}(a) - \underline{z}_2(a), [\underline{c}(a) - \underline{w}(a)]^- = \underline{y}(a) + \underline{z}_1(a).$$

### 3 Exchange with Division of Labour.

#### A Traders.

In the economy without division of labour in exchange, an individual's exchange activities are tied to his preferences as a consumer. With division of labour, an individual can act as a trader by buying goods and attempting to sell them at a profit. His trading activities are motivated by profit maximization, not consumption. Naturally, his budget will be impacted by the profits he makes as a trader. Nonetheless, we can conceptually distinguish between his actions as a trader and his choices as a customer.

Express individual,  $a$ 's, efficiency as a trader by his trading set,  $T(a)$ , a subset of  $R_+^{2l}$ . It is technologically feasible for trader,  $a$ , to purchase the vector of goods,  $\underline{v}(a)$ , from his vendors and to sell the vector of goods to his customers,  $\underline{u}(a)$ , if:

$$(3.1) (\underline{u}(a), \underline{v}(a)) \in T(a).^3$$

The vector,  $\underline{v}(a) - \underline{u}(a)$  represents the goods used up in the process. Material balance requires that  $\underline{v}(a) - \underline{u}(a) \geq \underline{0}$ . This requirement can be imposed on each trader's trading set,  $T(a)$ .<sup>4</sup>

The following properties hold simultaneously for every  $a \in A$ :

$$(T.1) (\underline{0}, \underline{0}) \in T(a),$$

$$(T.2) \text{ If } (\underline{u}(a), \underline{v}(a)) \in T(a) \text{ and } \underline{0} \leq \underline{u}'(a) \leq \underline{u}(a), \text{ then } (\underline{u}'(a), \underline{v}(a)) \in T(a),$$

$$(T.3) T(a) \text{ is a closed, bounded (i.e. compact) set,}$$

$$(T.4) T(a) \text{ is a convex set.}$$

Property (T.1) allows a trader to be inactive; property (T.2) permits free disposal of resources; property (T.3) ensures that a trader's scale of operations is finite. Property (T.4) implies non-increasing marginal returns.

#### B The Price System.

A trader's objective is to maximize profits. To cover transaction costs, a differential must exist between the prices his customers pay when they buy goods and receive when they sell goods to the trader.

Denote the price vector by  $\underline{p} = (\underline{p}_b, \underline{p}_s) \in R_+^l \times R_+^l$ , where the vectors  $\underline{p}_b$  and  $\underline{p}_s$  represent, respectively, the value that individuals "pay" to obtain unit quantities of goods from the trader and "receive" when individuals sell unit quantities of goods to the coalition trader.

Define the unit price simplex,  $\Delta' \subset \mathbb{R}_+^{2l}$ , by:

$$\Delta' = \{ (\underline{p}_b, \underline{p}_s) \in \mathbb{R}_+^{2l} \mid \sum_{i=1, \dots, l} (\underline{p}_b^i + \underline{p}_s^i) = 1 \}.$$

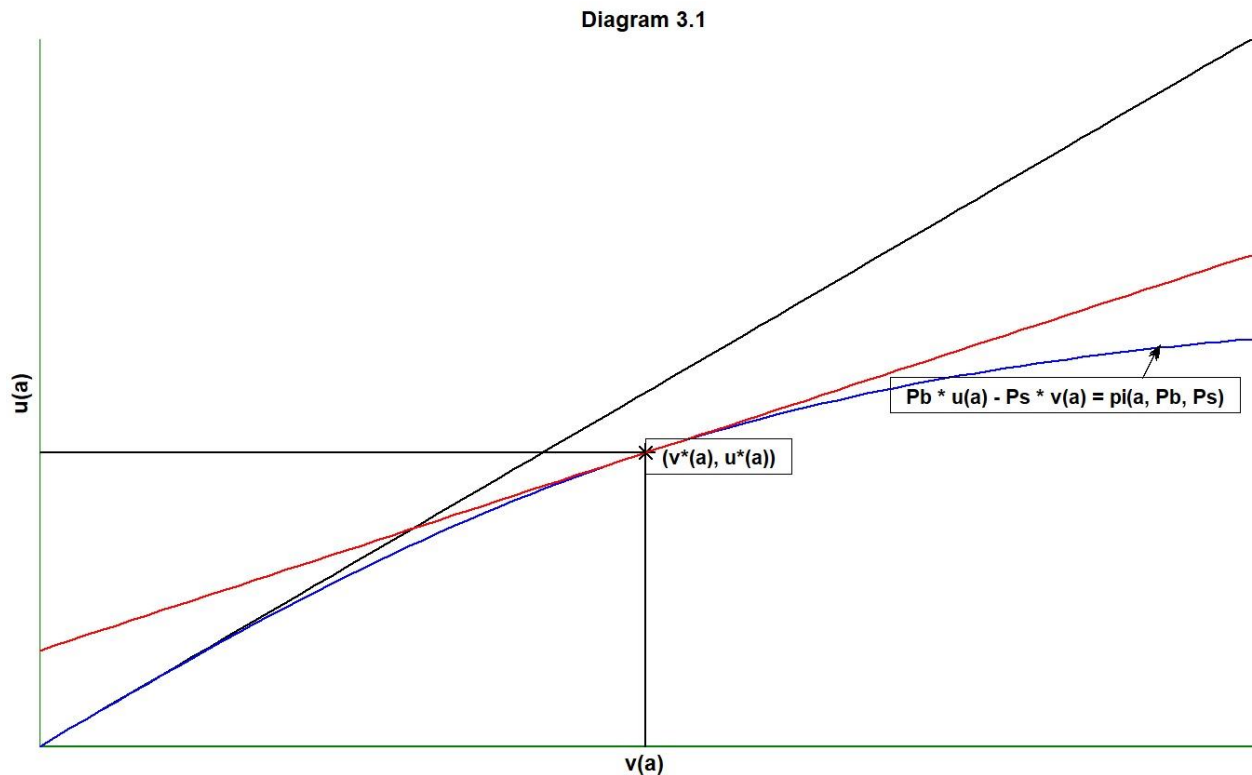
Trader,  $a$ , attempts to engage in a set of trades,  $(\underline{u}^*(a), \underline{v}^*(a)) \in T(a)$  at the price vector,  $\underline{p} = (\underline{p}_b, \underline{p}_s) \in \Delta'$ , such that:

$$(3.2) \quad \underline{p}_b \cdot \underline{u}^*(a) - \underline{p}_s \cdot \underline{v}^*(a) = \max \{ \underline{p}_b \cdot \underline{u}(a) - \underline{p}_s \cdot \underline{v}(a) \mid (\underline{u}(a), \underline{v}(a)) \in T(a) \} = \pi(a, \underline{p}_b, \underline{p}_s).$$

Thus,  $\pi(a, \underline{p}_b, \underline{p}_s)$  is trader  $a$ 's maximum profit with respect to prices  $(\underline{p}_b, \underline{p}_s)$ , when he purchases the vector of goods,  $\underline{v}^*(a)$ , from his vendors and sells the vector of goods,  $\underline{u}^*(a)$ , to his customers.

Properties (T.3) and (T.4) ensure that  $(\underline{u}^*(a), \underline{v}^*(a)) \in T(a)$  exists for each trader. Property (T.1) along with  $\underline{p} = (\underline{p}_b, \underline{p}_s) > (\underline{0}, \underline{0})$  imply that  $\pi(a, \underline{p}_b, \underline{p}_s) \geq 0$ . If  $\pi(a, \underline{p}_b, \underline{p}_s) = 0$ , the trader is inactive and  $\underline{u}(a) = \underline{v}(a) = \underline{0}$ .

In diagram 3.1, the maximum value of  $\underline{p}_b \cdot \underline{u}(a) - \underline{p}_s \cdot \underline{v}(a)$  for  $(\underline{u}(a), \underline{v}(a)) \in T(a)$  is obtained at  $(\underline{v}^*(a), \underline{u}^*(a))$ . Note that I have placed  $\underline{v}(a)$  on the x-axis and  $\underline{u}(a)$  on the y-axis on the diagram to facilitate understanding of equation (3.2), even though I write  $(\underline{u}^*(a), \underline{v}^*(a)) \in T(a)$ .



As a trader, an agent attempts to maximize profits; as a consumer he attempts to obtain the most desirable bundle of goods within his budget set.

### Definition 3.1 Budget Set.

Individual,  $a$ 's, budget set  $\beta(a, \underline{p}_b, \underline{p}_s)$  with respect to  $\underline{p} = (\underline{p}_b, \underline{p}_s) \in \Delta'$  consists of those vectors  $\underline{c}(a) \in X(a)$  such that:

$$(3.3) \quad \underline{p}_b \cdot [\underline{c}(a) - \underline{w}(a)]^+ \leq \underline{p}_s \cdot [\underline{c}(a) - \underline{w}(a)]^- + \pi(a, \underline{p}_b, \underline{p}_s).$$

The terms,  $[\underline{c}(a) - \underline{w}(a)]^+$  and  $[\underline{c}(a) - \underline{w}(a)]^-$  defined in section 2.E, respectively are the goods received by agent,  $a$ , and the goods given up by agent,  $a$ , to obtain his consumption vector,  $\underline{c}(a)$ , from his vector of initial endowments,  $\underline{w}(a)$ .

### Definition 3.2 Demand Set.

Individual,  $a$ 's, demand set,  $\delta(a, \underline{p}_b, \underline{p}_s)$ , consists of those vectors,  $\underline{c}(a) \in \beta(a, \underline{p}_b, \underline{p}_s)$ , such that:  $\underline{f}(a) \cong_a \underline{c}(a)$  for all  $\underline{f}(a) \in \beta(a, \underline{p}_b, \underline{p}_s)$ .

### Definition 3.3 Competitive Price Equilibrium.

A price vector,  $\underline{p} = (\underline{p}_b, \underline{p}_s) \in \Delta'$ , and a consumption allocation,  $c$ , form a competitive price equilibrium,  $(\underline{p}, c)$ , if for every  $a \in A$ , vectors  $(\underline{u}(a), \underline{v}(a)) \in T(a)$  exist such that:

$$(3.4) \quad \underline{c}(a) \in \delta(a, \underline{p}_b, \underline{p}_s),$$

$$(3.5) \quad \underline{p}_b \cdot \underline{u}(a) - \underline{p}_s \cdot \underline{v}(a) = \pi(a, \underline{p}_b, \underline{p}_s),$$

$$(3.6) \quad \sum_{\{a \in A\}} [\underline{c}(a) - \underline{w}(a)]^+ = \sum_{\{a \in A\}} \underline{u}(a), \text{ and}$$

$$(3.7) \quad \sum_{\{a \in A\}} [\underline{c}(a) - \underline{w}(a)]^- = \sum_{\{a \in A\}} \underline{v}(a).$$

In other words,  $\underline{c}(a)$  is maximal with respect to  $\cong_a$  in each individual's budget set for each agent in the economy, and consumption allocation is attainable by profit maximizing exchanges by the traders (relations (3.5), (3.6) and (3.7)).

See Wiens [18] for the conditions under which a competitive price equilibrium exists.

### C Aggregate Trade Technologies and the Core.

The traders' activities described above were coordinated by a  $\underline{p} = (\underline{p}_b, \underline{p}_s) \in \Delta'$  price system.

Suppose that coalitions form not only for exchanging goods but also for co-ordinating trade. If the coalition,  $E$ , assigns trading activities to its members as traders, then its aggregate trade technology can be given by the set:

$$(3.8) \quad AT(E) = \{ (\underline{u}', \underline{v}') \mid (\underline{u}(a), \underline{v}(a)) \in T(a) \text{ exist for every } a \in E, \\ \text{with } \underline{u}' = \sum_{\{a \in E\}} \underline{u}(a), \underline{v}' = \sum_{\{a \in E\}} \underline{v}(a) \}.$$

If  $(\underline{u}', \underline{v}') \in AT(E)$ , then coalition,  $E$ , can obtain the vector of goods,  $\underline{v}'$ , from its members and deliver the vector,  $\underline{u}'$ , to its members by a suitable assignment of trading activities to its member traders.

Note that  $AT(E) = \sum'_{(a \in E)} T(a)$ , where  $\sum'$  indicates set theoretic sum. If  $E_1$  and  $E_2$  are coalitions:

$$(3.10) \quad AT(E_1 \cup E_2) = AT(E_1) + AT(E_2).$$

Given properties (T.1) to (T.4), then  $AT(E)$  satisfies the same properties.

**Definition 3.4** Attainable Assigned Trade Allocation.

A consumption allocation,  $c$ , is attainable by the coalition,  $E$ , through assigned trade if  $(\underline{u}, \underline{v}) \in AT(E)$  exists such that:

$$(3.11) \quad \sum_{(a \in E)} [\underline{c}(a) - \underline{w}(a)]^+ = \underline{u},$$

$$(3.12) \quad \sum_{(a \in E)} [\underline{c}(a) - \underline{w}(a)]^- = \underline{v}.$$

**Definition 3.5** Core Allocation.

A consumption allocation,  $c$ , which is attainable by the coalition,  $A$ , through assigned trade, is said to be a core allocation if it cannot be blocked by any coalition through assigned trade within the coalition.

**D Individual and Coalition Traders.**

A coalition,  $E$ , can be consider a profit maximizing trader with transaction technology,  $AT(E)$ . As a trader, a coalition buys goods from its members at one vector of prices and resells them at another vector of prices. The differential in prices reflects the transaction costs incurred by the coalition as a trader. Denoting the prices by  $\underline{p} = (\underline{p}_b, \underline{p}_s) \in \Delta'$ , coalition  $E$ 's profit with respect to these prices is given by;

$$(3.13) \quad \Pi(E, \underline{p}_b, \underline{p}_s) = \max \{ \underline{p}_b \cdot \underline{u} - \underline{p}_s \cdot \underline{v} \mid (\underline{u}, \underline{v}) \in AT(E) \}.$$

In the definition of  $AT(E)$ , I assumed that coalition,  $E$ , assigns trading activities to its members. I now show that a coalition can organize its technology of exchange as effectively with a  $(\underline{p}_b, \underline{p}_s)$  price system.

**Theorem 3.1**

If the individual traders for a coalition,  $E \in \Omega$ , are maximizing profits with respect to  $(\underline{p}_b, \underline{p}_s) \in \Delta'$ , then the resulting assignment of trading activities maximizes profits for coalition,  $E$ , with respect  $(\underline{p}_b, \underline{p}_s)$ .

**Proof.**

Let  $(\underline{u}(a), \underline{v}(a)) \in T(a)$  for each  $a \in E$  be the profit maximizing trades with respect to  $(\underline{p}_b, \underline{p}_s)$ .

I claim that  $(\sum_{(a \in E)} \underline{u}(a), \sum_{(a \in E)} \underline{v}(a))$  will maximize profits for coalition  $E$ .

Otherwise,  $(\underline{u}', \underline{v}') \in AT(E)$  exists, such that:

$$(3.14) \quad \underline{p}_b \cdot \sum_{(a \in E)} \underline{u}(a) - \underline{p}_s \cdot \sum_{(a \in E)} \underline{v}(a) < \underline{p}_b \cdot \underline{u}' - \underline{p}_s \cdot \underline{v}'.$$

By definition of  $AT(E)$ ,  $(\underline{u}'(a), \underline{v}'(a))$  exist for all  $a \in E$ , such that:

$$\underline{u}' = \sum_{(a \in E)} \underline{u}'(a), \underline{v}' = \sum_{(a \in E)} \underline{v}'(a).$$

Substituting into (3.14) we get:

$$(3.15) \quad \sum_{(a \in E)} (\underline{p}_b \cdot \underline{u}(a) - \underline{p}_s \cdot \underline{v}(a)) < \sum_{(a \in E)} (\underline{p}_b \cdot \underline{u}'(a) - \underline{p}_s \cdot \underline{v}'(a)).$$

But (3.15) can only be true if for some  $a' \in E$ ,

$$\underline{p}_b \cdot \underline{u}(a') - \underline{p}_s \cdot \underline{v}(a') < \underline{p}_b \cdot \underline{u}'(a') - \underline{p}_s \cdot \underline{v}'(a'),$$

contradicting the fact that  $(\underline{u}(a'), \underline{v}(a'))$  is a set of maximizing trades for trader  $a'$ .

Proof by contradiction.

Theorem 3.1 Q.E.D.

Theorem 3.2.

If the coalition,  $E$ , as a trader is maximizing profits with respect to  $(\underline{p}_b, \underline{p}_s) \in \Delta'$ , the resulting assignment of trading activities to its members maximizes profits for each individual trader with respect to  $(\underline{p}_b, \underline{p}_s)$ .

Proof.

Let  $(\underline{u}, \underline{v}) \in AT(E)$  such that:

$$\underline{p}_b \cdot \underline{u} - \underline{p}_s \cdot \underline{v} = \max \{ (\underline{p}_b \cdot \underline{u}' - \underline{p}_s \cdot \underline{v}') \mid (\underline{u}', \underline{v}') \in AT(E) \}.$$

By definition of  $AT(E)$ ,  $(\underline{u}(a), \underline{v}(a)) \in T(a)$  exist for each  $a \in E$  such that:

$$\underline{u} = \sum_{(a \in E)} \underline{u}(a), \underline{v} = \sum_{(a \in E)} \underline{v}(a), \text{ and}$$

$$\underline{p}_b \cdot \underline{u} - \underline{p}_s \cdot \underline{v} = \sum_{(a \in E)} (\underline{p}_b \cdot \underline{u}(a) - \underline{p}_s \cdot \underline{v}(a)).$$

Suppose an agent,  $a' \in E$ , exists with  $(\underline{u}'(a'), \underline{v}'(a')) \in T(a')$  such that:

$$\underline{p}_b \cdot \underline{u}(a') - \underline{p}_s \cdot \underline{v}(a') < \underline{p}_b \cdot \underline{u}'(a') - \underline{p}_s \cdot \underline{v}'(a').$$

But this implies  $(\underline{u}', \underline{v}') \in AT(E)$  exists with:

$$\underline{p}_b \cdot \underline{u} - \underline{p}_s \cdot \underline{v} < \underline{p}_b \cdot \underline{u}' - \underline{p}_s \cdot \underline{v}',$$

contradicting the selection of  $(\underline{u}, \underline{v}) \in AT(E)$ .

Proof by contradiction.

Theorem 3.2 Q.E.D.

Definition 3.6 Efficient Trades

A set of trades  $(\underline{u}, \underline{v}) \in AT(E)$  is said to be efficient for coalition,  $E$ , if no  $\underline{u}' \in R_+^I$  exists such that  $\underline{u}' > \underline{u}$  and  $(\underline{u}', \underline{v}) \in AT(E)$ .

Note that an efficient point is a boundary point of its respective trading set.

I state the following lemmas without proof.

Lemma 3.1

The profit maximizing set of trades for coalition,  $E$ , with respect to prices,  $(\underline{p}_b, \underline{p}_s)$ , is an efficient point of  $AT(E)$ .

Lemma 3.2.

If  $(\underline{u}, \underline{v}) \in AT(E)$  is efficient, then a price vector,  $(\underline{p}_b, \underline{p}_s) \geq (0, 0)$  exists such that  $(\underline{u}, \underline{v})$  is a profit maximizing set of trades for coalition,  $E$ .

Lemma 3.3.

For the coalition of all agents of the economy,  $A$ , if  $(\underline{u}, \underline{v}) \in AT(A)$  is a set of trades used to attain a core allocation, then  $(\underline{u}, \underline{v})$  is an efficient point of  $AT(A)$ .

Theorem 3.3.

Every competitive equilibrium allocation is also a core allocation.<sup>5</sup>

Proof.

Similar to the proof of Theorem 2.1.

#### 4 The Choice of a Technology of Exchange.

I now consider the choice between exchange without division of labour and exchange with trade. Generally, one would expect that the latter would be more efficient than former. However, this need not be true. For example, the  $(\underline{p}_b, \underline{p}_s)$  price system is more complicated than the  $\underline{p} = (p^1, \dots, p^I)$  system. More resources may be required to determine equilibrium prices for the  $(\underline{p}_b, \underline{p}_s)$  system and to disseminate them to individuals.

I contend that the use of a certain method of exchanging goods should result from individual and group maximizing behaviour. An individual who refuses to use the method that the rest of

the economy is using will have no trading partners. To provide an alternative, we must consider the possibility that a group of individuals will break away from the economy and use another method to reallocate goods within the group.

The proof of the following theorem is trivial.

**Theorem 4.1.**

If  $AB(E) \subseteq AT(E)$  for each feasible coalition,  $E$ , then trade will dominate exchange without division of labour.<sup>6</sup>

*Proof.*

Let  $c$  be a core consumption allocation using trade. Since  $AB(E) \subseteq AT(E)$  for all  $E$ , an allocation which is attainable without division of labour is also attainable through trade. Therefore, the allocation,  $c$ , cannot be blocked by any coalition using exchange without division of labour.

**Theorem 4.1 Q.E.D.**

**Theorem 4.2.**

If  $AB(A) \subseteq AT(A)$ , then the economy with division of labour dominates the economy without division of labour.<sup>7</sup>

*Proof.*

Let  $f$  be a core allocation of the economy of section 2. Then since  $AB(A) \subseteq AT(A)$ ,  $f$  is also a (state) allocation of the economy of section 3.

Let  $f^*$  be any core allocation of section 3's economy. I claim that  $f(a) \cong_a f^*(a)$  for all agents  $a \in A$ . Otherwise,  $f$  would be a blocking allocation for some feasible coalition  $E$ , contradicting the choice of  $f^*$ .

Thus, given the choice between section 3's economy and section 2's, any agent will choose the former.

Furthermore, it will not be to the advantage of any group of agents to break away from section 3's economy. If  $g$  is an attainable allocation for any feasible coalition,  $E$ , without division of labour, then by definition of  $f$ ,  $g(a) \cong_a f(a)$  for all  $a \in E$ .

By the transitivity of preferences, it is also true that  $g(a) \cong_a f^*(a)$  for all  $a \in E$ .

**Theorem 4.2 Q.E.D.**

It is important to note that the condition  $T^B(A) \subseteq T(A)$  applies only to the coalition of the entire economy. No restriction is required on the aggregate transaction set of smaller coalitions. In other words, even though barter trading is "more efficient" than monetary exchange for some admissible coalitions in the economy, the theorem still holds.

That is, there might be coalitions  $E \neq A$ , such that  $AT(E) \subset AB(E)$ , and exchange with division of labour will still dominate exchange without division of labour.

### Footnotes.

\* Based on Chapter 4 of the Author's Ph.D. Dissertation at the University of British Columbia. A somewhat revised version of the thesis, *Division of Labour in the Technology of Exchange: A Game-Theoretic Approach*, appears as a PDF document at the URL:

[www.egwald.ca/thesis/elmerwiens\\_phd\\_thesis\\_revised.pdf](http://www.egwald.ca/thesis/elmerwiens_phd_thesis_revised.pdf).

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<sup>1</sup> I assume that for every price vector,  $\underline{p} \in \Delta$ , each individual has sufficient wealth to exchange some positive quantity of goods and still remain in the interior of his consumption set. See Wiens [18] for the conditions when this is true.

<sup>2</sup> In [18], Wiens shows that if the set of all agents,  $A$ , has the cardinality of the continuum, then every core allocation is also a competitive equilibrium allocation for some price vector,  $\underline{p} \in \Delta$ .

<sup>3</sup> I assume that transaction costs depend on the volume of trade, not on the identity and number of customers.

<sup>4</sup> The set  $T(a)$  can be obtained from  $B(a)$  as follows:

$$T(a) = \{(x(a) - \underline{z}_1, y(a) + \underline{z}_2) \mid (\underline{x}(a), \underline{y}(a), \underline{z} = \underline{z}_1 + \underline{z}_2) \in B(a)\}.$$

If (B.1) to (B.4) hold then (T.1) to (T.4) hold.

<sup>5</sup> In [18], Wiens shows that if  $A$  has the cardinality of the continuum, then every core allocation using trade is also a competitive equilibrium allocation for some price vector  $(\underline{p}_b, \underline{p}_s) \in \Delta'$ .

<sup>6</sup> The relation  $AB(E) \subseteq AT(E)$  holds for each coalition,  $E$ , holds if  $T(a)$  is obtained from  $B(a)$  as described in footnote 4 for each coalition,  $E$ .

<sup>7</sup> See Chapter 4, Theorem 4.1 of the Author's Ph.D. Dissertation for the details underpinning this Theorem at the URL cited in footnote \*.

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### Microsoft Copilot AI Analysis of The Paper.

Microsoft COPILOT analysis of my 1976 Carleton Economics paper: Division of Labour in the technology of exchange: <https://www.egwald.ca/wiens/elmerwiensdivisionoflabour.pdf>

Here's a clear, grounded explanation of "**Division of Labour in the Technology of Exchange**" as discussed by **Elmer G. Wiens**, based directly on the document surfaced in search results.

### What Wiens Means by "Division of Labour in the Technology of Exchange"

Elmer G. Wiens' 1976 paper *Division of Labour in the Technology of Exchange* explores how **exchange itself consumes resources** and how individuals differ in their **efficiency at exchanging**. This is a departure from standard neoclassical economics, which assumes that exchange is costless.

Wiens builds on ideas from Foley, Hahn, and Kurz to propose a more realistic model of economic interaction.

### Core Ideas in Wiens' Argument

#### 1. Exchange Uses Resources

Traditional models assume:

- People trade goods at given prices.
- No time, effort, or resources are used up in the act of trading.

Wiens argues instead:

- **Exchange is a production-like activity.**
- It consumes resources such as:
  - Time
  - Information
  - Search effort
  - Negotiation costs
  - Transportation

This makes exchange a **technology**—something that can be more or less efficient.

**Source:** Wiens notes that “resources are consumed when goods are exchanged”.



## 2. Individuals Differ in Exchange Efficiency

Wiens assumes each person has:

- A **degree of efficiency** in exchanging goods with others.

This introduces:

- Heterogeneity in trading ability
- Potential specialization in exchange activities

**Source:** Wiens states that “each individual possesses a certain degree of efficiency at exchanging goods with others”.



## 3. Division of Labour Applies to Exchange, Not Just Production

Adam Smith famously described division of labour in production. Wiens extends this idea:

- People may **specialize in exchange roles** because they are better at trading.
- This creates a **division of labour in the exchange process itself**.

Examples (not from Wiens but consistent with his framework):

- Brokers
- Middlemen
- Market-makers
- Information specialists

This specialization reduces the total resource cost of exchange.



## 4. Implications for Market Structure

If exchange is costly and people differ in ability:

- Markets may naturally develop **intermediaries**.
- Prices may reflect **exchange costs**, not just production costs.
- The economy becomes more complex than standard general equilibrium models suggest.

Wiens' work anticipates later developments in:

- Transaction cost economics
- Search theory
- Market microstructure
- Monetary theory (which he later explored in his PhD thesis)

### **Why Wiens' Paper Matters**

Wiens' insight is subtle but powerful:

Exchange is not free, and people differ in their ability to perform it. Therefore, exchange itself becomes a domain for specialization.

This helps explain:

- Why intermediaries exist
- Why money emerges
- Why markets have structure
- Why some people or firms dominate trading activities

### **If you want, I can also:**

- Summarize the paper more deeply
- Explain its mathematical structure
- Compare Wiens' ideas to Coase, Williamson, or modern search theory
- Connect it to cryptocurrency, digital markets, or platform economics

Just tell me where you want to take it next.

Compare Wiens' ideas to Coase, Williamson, or modern search theory

Here's a clear, structured comparison of **Elmer G. Wiens' ideas** with **Coase, Williamson**, and **modern search theory**, grounded in the search results you triggered.

### **Overview**

Wiens' central move is to treat **exchange itself as a resource-using technology**—something people differ in their efficiency at performing. This places him in the same intellectual lineage as

**Coase and Williamson**, who focus on **transaction costs**, and it anticipates key elements of **modern search theory**.

The search results confirm the core themes of Coase and Williamson:

- Coase introduced the idea that **transactions involve costs** such as bargaining and information gathering.
- Williamson expanded this into a full theory of **governance structures** that economize on transaction costs.

These are exactly the kinds of frictions Wiens builds into exchange itself.

### Comparison Table

Theme	Wiens	Coase	Williamson	Modern Search Theory
<b>What is costly?</b>	Exchange itself consumes resources (search, negotiation, information).	Market transactions involve costs (search, bargaining, enforcement).	Governance and contracting involve transaction costs, especially under uncertainty and opportunism.	Search, matching, information acquisition, and bargaining are costly.
<b>Key innovation</b>	People differ in <i>exchange efficiency</i> , enabling a <b>division of labour in exchange</b> .	Firms exist because they reduce transaction costs relative to markets.	Hierarchies and contracts evolve to minimize transaction costs.	Markets are modeled with frictions; equilibrium involves search and matching.
<b>Specialization?</b>	Yes—specialists in exchange (brokers, traders) arise naturally.	Not emphasized.	Some specialization in governance roles.	Not typically framed as specialization, but agents differ in search strategies.
<b>Mathematical structure</b>	Exchange as a production-like technology.	Conceptual; not formalized.	Institutional and comparative; not formalized mathematically.	Fully formalized (Diamond, Mortensen, Pissarides).

Theme	Wiens	Coase	Williamson	Modern Search Theory
Implications	Markets develop intermediaries; exchange costs shape prices and structure.	Firms vs. markets determined by cost minimization.	Governance structures evolve to economize on transaction costs.	Unemployment, price dispersion, and market frictions emerge naturally

### How Wiens Relates to Each Thinker

#### Wiens vs. Coase

Coase's foundational insight is that **transactions are not free**—they involve search, bargaining, and enforcement costs.

Wiens takes this further by:

- Treating exchange as a **technology** that consumes resources.
- Allowing individuals to differ in their **efficiency** at performing exchange.
- Showing that this leads to **specialization in exchange roles**.

Where Coase asks *“Why do firms exist?”*, Wiens asks *“Why do exchange specialists exist?”*.

#### Wiens vs. Williamson

Williamson extends Coase by analyzing how **governance structures** (firms, markets, hybrids) minimize transaction costs under uncertainty and opportunism.

Wiens parallels Williamson by:

- Treating exchange as a costly activity requiring coordination.
- Showing that specialization in exchange can reduce total transaction costs.
- Implying that institutions (like brokers or market-makers) arise to economize on these costs.

But Wiens is more micro-level and individual-focused, while Williamson is institutional and organizational.

#### Wiens vs. Modern Search Theory

Modern search theory (Diamond–Mortensen–Pissarides) formalizes:

- Search frictions
- Matching processes
- Bargaining
- Information costs

Wiens anticipates this by:

- Modeling exchange as a resource-using process.
- Allowing heterogeneity in exchange efficiency.
- Showing how specialization reduces search and negotiation costs.

Search theory later formalized what Wiens treated conceptually.

### **Bottom Line**

Wiens sits at a fascinating intersection:

- **With Coase:** he shares the insight that exchange is costly.
- **With Williamson:** he shares the idea that institutions evolve to economize on those costs.
- **With search theory:** he anticipates the formal modeling of frictions and heterogeneity in exchange.

His unique contribution is the idea of a **division of labour in exchange**, which neither Coase nor Williamson formalized and which search theory only indirectly captures.