Review of Disequilibrium in Markets:

The Micro-Foundations

Elmer G. Wiens

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Dr. R. Evans

1. Introduction

Axel Leijonhufvud in On Keynesian Economics and the Economics of Keynes posits that while in general equilibrium models, supply and demand are functions of prices, in "Keynesian" models supply and demand are functions of income and interest rates (51). This paper reviews attempts to lay a micro-foundation for "Keynesian" models using the concept of disequilibrium in markets.

Section 1 defines the concept of disequilibrium in markets with a model of an exchange economy. The model assumes that goods have been allocated to the agents, and / or can be produced by agents. It examines the issue of how the goods will be re-distributed through exchange with other agents when market prices do not equate the demand and supply of goods instantaneously, as in the general equilibrium models. Agents could be constrained selling or buying goods and / or choose to limit production, when markets do not clear demand with supply. With the model, we attempt to lay the micro-foundations of transactions arising out of market disequilibrium.

2. The Micro-Foundations of Disequilibrium

The Economy of "n" Markets and "m" Agents.

Assume the economy consists of:

1. n markets for goods labelled with the index $i = 1 \ldots n$,
2. m economic agents labelled with the index $j = 1 \ldots m$,
3. one good, "money," is a medium of exchange and unit of account.

At the start of the current time period, the economy's prevailing relative price vector $p = (p_1, \ldots, p_n)$, in terms of the unit of account. The economy's agents maximize their objective functions through exchange at these prevailing prices, subject to their transaction budget and goods transaction balance constraints. If the k^{th} good is money, p_k is the borrowing / lending interest rate for one unit of money for one time period. If the c^{th} good is a capital item, p_c is its rental rate for one unit for one time period. If the l^{th} good is labour, p_l is the wage rate per unit.

Formally, agents attempt to maximize their objective functions subject to constraints:

$\max f_j(x_1^j, \ldots, x_n^j)$, the j^{th} agent's objective function,

such that $\sum_{i=1}^{n} p_i x_i^j = 0, j = 1 \ldots m$, the j^{th} agent's transaction budget constraint,

where $x_i^j > 0$ if the i^{th} good is demanded by agent j,

$x_i^j < 0$ if the i^{th} good is supplied by agent j,

$x_i^j = 0$ if the i^{th} good is not transacted by agent j,

and $\sum_{j=1}^{m} x_i^j = 0, i = 1 \ldots n$, the i^{th} market's clearing condition

For intertemporal reasons, assume that at the start of each time period an agent is endowed with some goods, like labor (income) and money, that can be sold or used during the current period,
i.e. let:

$\cancel{x}^j = (\cancel{x}_1^j, \ldots, \cancel{x}_n^j)$, be the j^{th} agent's endowment vector of goods.

Also, let the inventory of goods carried over from the previous time period be:

$ix^j = (ix_1^j, \ldots ix_n^j)$,

Namely for the j^{th} agent, it is the vector of non-perishable goods left over at the end of the previous time period, plus repaid money loaned plus interest and returned capital goods to the j^{th} agent.

If agent j is a producer of goods, let:

$px^j = (px_1^j, \ldots, px_n^j)$, be the current period's vector of production by the agent.

For non-producing agents, $px^j = (0, \ldots, 0)$

Then, the vector of goods available for agent j to sell during the current time period is:

$\underline{x}^j = (\underline{x}_i^j, \ldots \underline{x}_n^j) = \cancel{x}^j + ix^j + px^j$, i.e. endowments + inventories + production.

Finally, the agent's goods transaction balance constraints for the current time period are:

$\underline{x}_i^j + x_i^j \geq 0, i = 1 \ldots m$.

If agent j buys good i, x_i^j is a positive amount. If agent j sells good i, then x_i^j is a negative amount

Assume that relative prices did not adjust instantaneously to equate demand and supply in all markets at the end of the previous time period; that market clearing is a gradual process. Thus, agents might discover they are unable to buy and / or sell all they want (consistent with maximization of their objective functions subject to the constraints) in a given market at the prevailing relative prices during the current time period

The j^{th} agent can be considered to solve the following Static Recursive Algorithm while undertaking transactions. This procedure is a generalization of Robert Clower's Dual Decision Hypothesis set out in "The Keynesian Counter-Revolution: A Theoretical Appraisal."

<u>The Static Decision Algorithm for j^{th} Agent</u>.

1. Start with the price vector $p = (p_1, \ldots, p_n)$, and the agent's objective function, and the transaction budget constraint and goods transaction balance constraints in the constraint set.
2. Maximize the objective function subject to the relations in the constraint set to determine the desired quantities x_i^j, $i = 1 \ldots n$.
3. Is the agent constrained in any (further) market i?
    a. If yes, add the effective transaction constraint $x_i^j \leq \dot{x}_i^j$ or $x_i^j \geq \dot{x}_i^j$ to the constraint set depending on whether the agent is constrained as a buyer or seller. Thus, $\dot{x}_i^j$ is the amount the agent is actually able to transact on the i^{th} market. (It is negative if sold.)
    Go to step 2.
    b. If no, label the resulting quantities bought or sold as $\ddot{x}_i^j$, $i = 1 \ldots n$.

The initial solution of the algorithm, when only the transaction budget and goods transaction balance constraints are in the constraint set, is denoted by the vector $x^j = (x_1^j, \ldots, x_n^j)$. It is called the notional demand / supply vector for the j^{th} agent.

The solution vector to the complete recursive algorithm, $\ddot{x}^j = (\ddot{x}_1^j, \ldots, \ddot{x}_n^j)$, is called the effective demand / supply vector of the j^{th} agent.

If the agent's notional demand or supply is constrained on a given market, the result on his effective demand in other markets will of course depend on his objective function. We list the following outcomes as being reasonable.

1. Consider the effect when the j^{th} agent can buy only $\ddot{x}_i^j < x_i^j$ on the i^{th} market.

If some of his demand "spills-over" into his demand for good h, we could get $\ddot{x}_h^j > x_h^j$.

Alternatively, if the agent decides to produce and sell less of a good s, being limited in the use of factor i, we get $\ddot{x}_s^j > x_s^j$.

2. Consider the effect when the j^{th} agent can sell only $\ddot{x}_i^j > x_i^j$ on the i^{th} market (a smaller negative number is greater than a larger negative number).

An agent who is a producer of good i might reduce purchases of good q, a factor in production; we could get $\ddot{x}_q^j < x_q^j$.

Alternatively, in a market r where the agent is also a buyer, we could get $\ddot{x}_r^j < x_r^j$, since to satisfy the transaction budget constraint, the agent will use less of some goods.

All agents participate in their Static Decision Algorithm during the current time period. At the end of the current time period, all markets are assumed to have cleared. The ability of agents to transact their desired amounts could depend on when they engage in the market during the current time period. The notional and effective amounts demanded and supplied by each agent in each market are net amounts at the prevailing relative prices.

Market clearing, total amount bought equals total amount sold, of effective demand / supply implies that in all markets i:

$$\ddot{x}_i = \sum_{j=1}^{m} \ddot{x}_i^j = 0, \; i = 1 \ldots n. \quad \text{so that} \quad \ddot{x} = (\ddot{x}_1, \ldots, \ddot{x}_n) = (0, \ldots, 0).$$

<u>The Dynamic Adjustment Mechanism for Prices</u>.

Measure disequilibrium in the economy at the end of the current period by the differences between notional and effective demand / supply. Postulate that the prices prevailing in the next time period will reflect these differences.

In each market, divide agents into groups depending on whether they are buyers, sellers, or neither. Formally, for the i^{th} market let:

$$J = (1 \ldots m), \text{ the integer index set of all agents.}$$

$J_i^B = j \in J$ such that $\ddot{x}_j^j > 0$, the index set of agents buying good i,
$J_i^S = j \in J$ such that $\ddot{x}_j^j < 0$, the index set of agents selling good i,
$J_i^N = j \in J$ such that $\ddot{x}_j^j = 0$, the index set of agents not transacting good i.

$$J = J_i^B \cup J_i^S \cup J_i^N.$$

For the i^{th} market, the total amount of good i transacted (total amount bought = total amount sold) is:

$$\ddot{x}_i^T = \text{sum } (\ddot{x}_i^j) \text{ for } j \in J_i^B = \text{-sum } (\ddot{x}_i^j) \text{ for } j \in J_i^S \geq 0.$$

For a market i, consider agent $j \in J_i^B$, a buyer of good i. If $(x_i^j - \ddot{x}_i^j) > 0$, the agent's notional demand is greater than the agent's effective demand. If as a result of a "spill-over" effect $(x_i^j - \ddot{x}_i^j) < 0$, the opposite is true, and the agent's excess demand is effectively zero.

Define the effective excess demand for the i^{th} market by:

$$e_i^D = \text{sum } \{\max [(x_i^j - \ddot{x}_i^j), 0]\} \text{ for agents } j \in J_i^B.$$

If at price p_i, buyers in total want to buy more than their transacted amounts, $e_i^D > 0$. Here, one might expect upward pressure on the future price for good i, with an increase in the total amount of good i transacted in the next time period.

Now consider agent $j \in J_i^S$, a seller of good i. If $(x_i^j - \ddot{x}_i^j) < 0$, the agent's notional supply is greater than the agent's effective supply, in absolute value. Conversely, with "spill-over" effects, $(x_i^j - \ddot{x}_i^j) > 0$, the opposite is true, and here the agent's excess supply is effectively zero.

Define the effective excess supply for the i^{th} market by:

$$e_i^S = \text{sum } \{\min [(x_i^j - \ddot{x}_i^j), 0]\} \text{ for } j \in J_i^S.$$

If at price p_i, sellers in total want to sell more than their transacted amounts,

$e_i^S < 0$. Here, one might expect downward pressure on the future price for good i with a decrease in the total amount of good i transacted in the next time period.

The excess transaction vector is $e^T = (e_1^T, \ldots, e_n^T)$, with components $e_i^T = e_i^D + e_i^S$.

We are assuming that at the end of the current time period, buyers and sellers can negotiate or set the future price $\underline{p}_i$ that will prevail in the next time period. We posit a price adjustment mechanism for the i^{th} market, in which transactions have occurred, of the form:

$$\underline{p}_i = p_i + \alpha_i \left(e_i^D / \ddot{x}_i^T\right) + \beta_i \left(e_i^S / \ddot{x}_i^T\right),$$

We specify two positive adjustment coefficients, one for effective excess demand and one for effective excess supply.

Since the excess transaction ratios, $(e_i^D / \ddot{x}_i^T)$ and $(e_i^S / \ddot{x}_i^T)$, are, respectively, non-negative and non-positive per definitions, $\alpha_i > 0$ and $\beta_i > 0$.

At the start of the current time period, we had a relative price vector $p = (p_1, \ldots, p_n)$, that failed to clear all markets. At the start of the next period, we will have a relative price vector $p = \underline{p} = (\underline{p}_1, \ldots, \underline{p}_n)$, again with no guarantee that it will clear all markets.

3. Employment and Income

It appears that Don Patinkin, Money, Interest, and Prices, was the first person to show that disequilibrium in one market can "spill-over" and cause disequilibrium in another market. Suppose that a representative firm is unable to sell all it desires at the existing prices on a given market, when it regards profit maximization as being constrained only by its production function. The profit maximization problem then becomes to select the minimum quantity of labor necessary to produce the output quantity that it can sell, Herschel Grossman and Robert Barro in "A General Disequilibrium Model of Income and Employment" (85).

Thus, the effective demand for labor will be less than the notional demand for labor, and if prices do not adjust rapidly enough, the economy will have excess supply in the labor market. That is, the economy will have involuntary unemployment, even though the existing wage rate is the notional equilibrium wage rate. Further analysis suggests that real wages may move pro-cyclically even with excess supply of labor, a conjecture supported by Ronald Bodkin's empirical studies in "Real Wages and Cyclical Variations in Employment."

In "The Keynesian Counter-Revolution: A Theoretical Appraisal," Clower uses a disequilibrium choice theoretic framework as a basis in deriving Keynes' consumption function. In the classical analysis, the representative household regards utility maximization as being subject only to the budget constraint. This implies that the notional consumption function does not depend on income, because income (amount of labor it will supply) and consumption are chosen simultaneously.

If labor is in excess supply, the household must include the labor constraint when it maximizes utility. Thus, the effective consumption function will have effective income (i.e. the amount of labor it can sell) as one of its arguments. But this implies that effective demand in the commodity market will be less than the notional demand. That is, the excess supply in the labor market has contributed to excess supply in the commodity market.

Grossman and Barro analyze further the effect of disequilibrium arising out of excess demand. Using the disequilibrium choice theoretic framework, it follows that if there is excess demand for labor, the firm will either increase its demand for capital and/or produce less. If there is excess demand for commodities, the household will either increase its money balances and/or decrease its supply of labor.

The conclusions obtained by these authors are consistent with our model in the subsection The Static Decision Algorithm for j^{th} Agent of Section 2.

4. Prices and Interest Rates

Grossman, "Money, Interest, and Prices in Market Disequilibrium," uses the disequilibrium choice theoretic framework to investigate "the appropriate specification of the market pressures which make prices change" in a disequilibrium context. His definition of effective demand is somewhat similar to our definition in section 2. He focuses on the degree to which effective demand constraints in one market may increase demand in other markets.

His economy consists of three markets—commodities (y) with price p, debt (b) with interest rate r, and money (n). The variables (y, b, n) represent notional amounts; the variables ($\underline{y}$, $\underline{b}$, $\underline{n}$) represent effective amounts.

If demand in the commodity market is constrained ($\underline{y} < y$), demand will either "spill-over" into the debt market ($\underline{b} > b$) and/or into the money market ($\underline{n} > n$).

Alternatively, if demand in the debt market is constrained ($\underline{b} < b$) demand will "spill-over" into the commodity market ($\underline{y} > y$) and/or into the money market ($\underline{n} > n$).

The behavioral equations for the j[th] agent specified by Grossman are:

Case 1.

$$\underline{b}_j = b_j + \alpha_j (y_j - \underline{y}_j),$$
$$\underline{n}_j = n_j + (1- \alpha_j) (y_j - \underline{y}_j),$$
$$\alpha_j = \alpha_j(r, p, \underline{y}_j);$$

Case 2.

$$\underline{y}_j = y_j + \beta_j (b_j - \underline{b}_j),$$
$$\underline{n}_j = n_j + (1- \beta_j) (b_j - \underline{b}_j),$$
$$\beta_j = \beta_j (r, p, \underline{b}_j).$$

The coefficients α_j and β_j are the "spill-over" coefficients of the j[th] agent.

Grossman shows that in Case 1., if the 'spill-over" is entirely into the money market ($\alpha_j = 0$), changes in the interest rate are consistent with the dynamic loanable funds theory (agents will hold money for liquidity). If the "spill-over" is entirely into the debt market ($\alpha_j = 1$), changes in the interest rate are consistent with the dynamic liquidity preference theory (agents will forego interest on money to buy secure price-protected assets).

If in Case 2., if the "spill-over" is entirely into the commodity market ($\beta_j = 1$), changes in the price level are consistent with the dynamic quantity theory of money (the price level p is directly proportional to the money supply). If the "spill-over" is entirely into the money market ($\beta_j = 0$), changes in the price level are consistent with the dynamic expenditure theory (agents will consume a proportion of their money holdings).

The above analysis agrees with William Baumol's conjecture in "Stocks, Flows and Monetary Theory" that the rate of interest will be determined by the market (debt or money) which returns to partial equilibrium the quickest in response to a disequilibrating event.

5. Investment

According to the neo-classical theory of demand for investment, a profit maximizing firm will desire an optimal stock of capital, based on its state of expectations about the prevailing economic conditions. If its capital stock is less than optimal, it will invest. Conversely, if its capital stock is greater than optimal, it will disinvest.

The income accelerator theory of investment demand, on the other hand, regards output as exogenously determined, with investment an increasing function of changes in the level of output.

In "A Choice-Theoretic Model of an Income-Investment Accelerator," Grossman provides "a reconciliation of the neoclassical and accelerator theories of investment demand." His approach is similar to those we have encountered in Sections 3. and 4., although here we are operating in an intertemporal context. If the market for output does not clear at the existing price, output to a profit maximizing firm is no longer a decision variable. Thus, effective "investment demand will become a function of the level of the level of output which it will be able to sell" (634).

Grossman shows that the assumption of static expectations about the output constraint implies a gradual (flexible) income-investment accelerator. On the other hand, the assumption of static expectations about the rate of change of constrained output implies an instantaneous accelerator relationship. Grossman concludes that profit maximization implies that if the output market is in disequilibrium, the income-accelerator approach is the correct theory of investments demand; if the output market is in equilibrium given the state of expectations, the neoclassical approach is the correct theory of investment demand.

6. Conclusions

In The Economy of "n" Markets and "m" Agents subdivision of section 2., we described the constraints that confront the agents attempting to maximize their objective functions, subject to a fixed vector of relative disequilibrium prices already in effect at the start of the current time period. As the agents buy and sell goods, they must satisfy their transaction budget and goods transaction balance constraints. The amounts demanded and supplied by agents of each good subject to these criteria is called the agents' notional demand for or supply of that good.

If the relative prices at which agents transact is not an equilibrium price vector, some agents will be unable to buy or sell their notional amounts in a given market. The Static Decision Algorithm for j^{th} Agent subdivision provides a process by which agents maximize their objective functions subject to the additional effective transaction constraints arising if the prevailing relative prices are not notional equilibrium prices. The result of this process for each agent is a vector of amounts actually transacted, the agent's effective demand and supply of goods.

In Section 3., Employment and Income, Section 4., Prices and Interest Rates, and Section 5., Investment, we exhibited how numerous recently published papers (1972) are based on a choice theoretic model whereby market disequilibrium may constrain the desired transactions of some agents. In each case, the authors of these papers particularized the more general disequilibrium model described in Section 2.

Neoclassical equilibrium analysis assumed that markets always clear with prices adjusting virtually instantaneously to equate demand and supply. When this assumption is dropped, "Keynesian" theories of economics are found to be based on a choice theoretic model which permits the actual demand and/or supply amounts transacted to be constrained.

In A Dynamic Adjustment Mechanism for Prices subsection of Section 2, we provided a detailed model of price adjustment arising out of disequilibrium in markets. The rate at which the price adjusts in the i^{th} market depends on two adjustment coefficients, α_i and β_i, and the excess transaction ratio e_i^T / $\ddot{x}_i^T$—the ratio of excess demand or supply to the total amount actually transacted. The speed at which the price for the i^{th} market adjusts depends on the values of α_i and β_i. However, we did not specify the price adjustment process, although negotiations on prices between individual buyers and sellers seems plausible.

In "A Theory of Monopolistic Price Adjustment," Barro states that "optimal price adjustment depends on certain 'institutional' characteristics under which trading occurs." Furthermore, "the response of prices to disequilibrium is essentially a monopolistic phenomenon (17). In "On Price Adjustment without an Auctioneer," Franklin Fisher extended his "model of a single market in which firms quoted individual prices and consumers searched for low quotations" (1). His research goal is to include quantity constraints and "spill-over" effects into a more formal procedure by which new (equilibrium?) relative prices are determined from trading by agents at disequilibrium relative prices.

It would appear that the concept of effective demand and supply arising out of disequilibrium will eventually have important consequences in many areas of theoretical and empirical research in economics.

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