DIVISION OF LABOUR IN THE TECHNOLOGY OF EXCHANGE

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1. Introduction

A. The traditional neo-classical models of an economy assume that no resources are used up when goods are exchanged. An individual is distinguished by his preferences and by his initial endowment of goods. He exchanges goods with respect to a parametric vector of prices to obtain the most preferred bundle of goods within his budget set.

Following Foley [6], Hahn [7], and Kurz [11] let us suppose that resources are consumed when goods are exchanged. Suppose also that each individual possess a certain degree of efficiency at exchanging goods with others. For example, individuals might have different abilities at searching for potential trading partners, at bargaining, or at transporting goods to and from the location where goods are physically transferred from the possession of one individual to another. We will represent an individual's ability at executing exchanges by his transaction technology. It describes all his feasible exchanges and their attendant resource costs.

B. Under these assumptions, we will describe two ways of organizing exchange activities.

The first method does not permit specialization or division of labour in the technology of exchange. Each individual is required to execute his own exchanges and to bear directly any transaction costs incurred. His desired exchanges will be based on his preferences, his existing stock of goods, his beliefs about the prevailing exchange ratios between goods, and his transaction technology.

The second method permits specialization. An individual is permitted to execute exchanges on behalf of others. He may act as a trader by buying goods from some individuals and reselling them to others. By acting as a trader, an individual hopes to consume a more desirable bundle of goods than he could have if he only exchanged goods on his own behalf, or, if he permitted some trader to execute his exchanges. Competition between traders will ensure that only the relatively efficient individuals act as traders.

C. Two equilibrium concepts, the core and the set of competitive price allocations, are considered for each method of exchanging goods. The core of an economy is the set of outcomes that no coalition can improve upon. A core allocation is a distribution of goods which is acceptable to every individual and to every coalition or group of individuals. Its definition is "institutionally free" in the sense that it does not depend on the existence of a parametric list of prices.

It is well known that if exchange is costless each competitive equilibrium allocation, the distribution of goods among individuals after exchanging goods at equilibrium prices, belongs to the core. For economies with a finite number of individuals, the core is generally larger than the set of competitive allocations. Debreu and Scarf [5] have shown that as the number of individuals "gets large" the core "shrinks" to the set of competitive allocations. The "equivalence" between the core and the set of competitive allocations has been demonstrated by Aumann [1] for a pure exchange economy and Hildenbrand [9] for a coalition production economy. The last two results depend on the assumption that an individual is insignificant in the sense that his presence does not significantly affect the outcome of exchanging goods.

D. In this paper we will deduce what the structure of prices must be, for each method of exchanging goods, such that a competitive equilibrium allocation is also a core allocation.

When division of labour is not permitted, an individual's budget set depends both on his transaction technology and on the ratios at which goods are exchanged. If individuals have different transaction technologies, they will also have different effective exchange ratios even though the prevailing exchange ratios are consistent and can be reduced to one price per good.

With traders, on the other hand, both buying and selling prices are required for each good. The differential between these prices reflects the transaction costs borne by the traders. Competition between traders will ensure that a consistent set of buying and selling prices will prevail. An individual's budget set depends on these prices and on the profit he earns as a trader with respect to these prices. An individual will only operate as a trader if his profit is non-negative.

E. We will use the theory of the core for two other purposes. First, we derive aggregate exchange technologies from individual transaction technologies. We show that a coalition can organize its member traders as effectively with a price system as it can be assigning exchange tasks to traders. In fact, profit maximization on the part of a coalition is equivalent to profit maximization on the part of individual traders. Second, we use the theory of the core to model the choice of the technology of exchange. We show that exchange with division of labour will be chosen instead of exchange without division of labour if for each coalition the aggregate transaction set of the latter is a subset of the aggregate transaction set of the former.

F. Notation

A = the set of all individuals in the economy

Ω = the set of feasible coalitions or subsets of A

R^{ℓ}_{+} = the non-negative orthant of the Euclidian space of dimension ℓ - called the commodity space.

ℓ = the number of commodities or goods.

w(a) = initial endowment of individual $a \epsilon A$, a vector in R^{ℓ}_{+}, i.e. $w(a) \epsilon R^{\ell}_{+}$.

X(a) = consumption set of individual $a \epsilon A$, a convex subset of R^{ℓ}_{+}.

c(a) = final consumption bundle of individual $a \epsilon A$, a vector in X(a).

$\precsim_a$ = individuals a's preference ordering defined on X(a). This ordering is transitive, reflexive, continuous, convex, and complete. From $\precsim_a$ we also define the ordering $\prec_a$ by $s \prec_a t$ if $s \precsim_a t$ but not $t \precsim_a s$.

2. Exchange Without Division of Labour

A. Transaction sets and exchange.

Express individual a's efficiency at exchanging goods by his transaction set B(a) which we postulate to be a subset of $R_{+}^{3\ell}$. The exchange of the vector $y(a)\epsilon R_{+}^{\ell}$ for the vector $x(a)\epsilon R_{+}^{\ell}$ is said to be technologically feasible for individual a if a vector $z(a)\epsilon R_{+}^{\ell}$ exists such that

$$(x(a),y(a),z(a))\ \epsilon B(a). \tag{2.1}$$

Vector z(a) represents the quantities of goods needed by a to complete the transaction. If he is successful then his resulting consumption bundle c(a) is given by

$$c(a) = w(a)+x(a)-y(a)-z(a). \tag{2.2}$$

Relations (2.1) and (2.2) express the constraints we place on exchange in this economy. An individual's exchanges are constrained both by his transaction set and by goods in his possession.

We assume that the following properties hold for every $a\epsilon A$.

(B.1) $0\ \epsilon\ B(a)$.

(B.2) If $(x(a),\ y(a),\ z(a))\ \epsilon\ B(a)$ then $x'(a)\leq x(a)$, $z'(a)\geq z(a)$ implies that $(x'(a),y(a),z'(a))\ \epsilon\ B(a)$.

(B.3) B(a) is a closed, bounded, set.

(B.4) B(a) is a convex set.

Property (B.1) admits no exchange; property (B.2) admits free disposal of resources. Property (B.3) ensures that an individual's scale of operations is finite; property (B.4) implies non-increasin marginal returns.

Definition 2.1 Allocation of goods

An allocation of goods, denoted by c, is a distribution of goods among the individuals in the economy where c(a), the vector assigned to individual a, is an element of X(a).

Definition 2.2 Attainable Allocation

An allocation c is said to be attainable by a coalition $E \in \Omega$ if for each $a \in E$ vectors x(a), y(a), and z(a) exist in R^{ℓ}_{+} such that (2.1) and (2.2) are satisfied and

$$\sum_{a \in E} c(a) = \sum_{a \in E} w(a) - \sum_{a \in E} z(a) \quad (2.3)$$

The last equation is coalition E's material balance equation. Notice that (2.2) and (2.3) imply that

$$\sum_{a \in E} x(a) = \sum_{a \in E} y(a) \quad (2.4)$$

In other words, the exchanges within a coalition must be mutually consistent.

B. The core

The core or the set of core allocations is based on the following rationality postualte. Let c be an allocation which is attainable by the coalition A. We say that c is blocked by a coalition E if an allocation h exists attainable by E such that

$$c(a) \precsim_a h(a) \quad \text{for all } a \in E$$

and $c(\bar{a}) \prec_{\bar{a}} h(\bar{a})$ for some $\bar{a}\epsilon E$.

Definition 2.3 Core allocation

An allocation c which is attainable by A and which cannot be blocked by any coalition E is said to be a core allocation.

The core is the set of all core allocations.

C. The price system.

Consider a pure exchange economy where exchange does not consume real resourecs. With ℓ goods in the economy, there will be $\ell(\ell-1)/2$ exchange ratios between goods. If arbitrage is effective or assumed, it is possible to reduce these exchange ratios to a set of ℓ-1 relative prices, denoted by $p\epsilon R^{\ell}_{+}$, where any good can be the numeraire.

If exchange consumes real resources, things are more complicated. We have mentioned that the effective exchange ratios faced by an individual in this economy depend not only on the ratios at which quantities are transferred from the possession of one individual to another, but also on the individual's transaction set. If individuals have different transaction sets, then their effective exchange ratios will differ as well.

Exchanging y(a) for x(a) consumes z(a) in resources. The vector z(a) is made up of goods from a's initial endowment and/or goods obtained from others during the process of exchange. If we denote these quantities by $z_1(a)$ and $z_2(a)$ respectively, where $z(a) = z_1(a)+z_2(a)$, then in effect individual a gives up the vector $y(a)+z_1(a)$ to obtain for consumption the vector $x(a)-z_2(a)$

Let the unit price simplex $\Delta \subset R^{\ell}_{+}$ be defined by $\Delta = \{p \epsilon R^{\ell}_{+} \mid \sum_{i=1}^{\ell} p^i = 1\}$, where p represents a set of ℓ-1 relative prices.

<u>Definition 2.4</u> <u>Budget set</u>.

Individual a's budget set $\beta(a,p)$ with respect to a price vector $p \epsilon \Delta$ is given by

$$\beta(a,p)=\left\{(x(a),y(a),z(a)) \;\middle|\; \begin{array}{l} a)\,(x(a),y(a),z(a))\epsilon B(a) \\ b)\,w(a)+x(a)-y(a)-z(a)\epsilon X(a) \\ c)\,p\cdot x(a) \leq p\cdot y(a) \end{array}\right\}$$

Part c) of the definition means that the value received is less than or equal to the value given up in exchange.

<u>Definition 2.5</u> <u>Demand set</u>.

Individual a's demand set $\delta(a,p)$ with respect to $p \epsilon \Delta$ is given by

$$\delta(a,p)=\{(x(a),y(a),z(a))\epsilon\beta(a,p) \mid \text{for every } (x'(a),y'(a),z'(a)) \text{ in } \beta(a,p) \text{ we have } w(a)+x'(a)-y'(a)-z'(a) \precsim_a w(a)+x(a)-y(a)-z(a)\}.$$

<u>Definition 2.6</u> <u>Competitive price equilibrium</u>.

The pair (p,c) forms a competitive price equilibrium where $p \epsilon \Delta$ if vectors x(a), y(a), z(a) exist for all $a \epsilon A$ such that

$$c(a) = w(a)+x(a)-y(a)-z(a) \qquad (2.7)$$

$$(x(a),y(a),z(a)) \in \delta(a,p), \text{ and} \tag{2.8}$$

$$\sum_{a\in E} c(a) = \sum_{a\in E} w(a) - \sum_{a\in E} z(a). \tag{2.9}$$

The allocation c is called a competitive equilibrium allocation.

See [18] for the conditions under which a competitive price equilibrium exists.

The proof of the following theorem is based on Hildenbrand [9].

<u>Theorem 2.1</u>

Every competitive equilibrium allocation is also a core allocation.

<u>Proof</u> Let c be a competitive equilibrium allocation with respect to a price vector $p \in R^{\ell}_{+}$. Suppose it is not a core allocation. Then an allocation h exists attainable by some coalition E blocking c. Formally, $x'(a), y'(a), z'(a) \in R^{\ell}_{+}$ exist such that

$$h(a) = w(a)+x'(a)-y'(a)-z'(a) \quad \text{all } a\in E, \tag{2.10}$$

$$(x'(a),y'(a),z'(a)) \in B(a) \quad \text{all } a\in E, \tag{2.11}$$

$$\sum_{a\in E} h(a) = \sum_{a\in E} w(a) - \sum_{a\in E} z'(a), \tag{2.12}$$

$$c(a) \precsim_a h(a) \quad \text{all } a\in E, \tag{2.13}$$

$$c(\bar{a}) \prec_{\bar{a}} h(\bar{a}) \quad \text{some } \bar{a}\in E. \tag{2.14}$$

But $c(a) \in \delta(a,p)$ for all $a\in E$ plus (2.13) and (2.14) imply

$$p.x'(a) \geq p.y'(a) \quad \text{all } a\in E, \text{ and}$$

$$p.x'(\bar{a}) > p.y'(\bar{a}).$$

The last two inequalities imply

$$\sum_{a\epsilon E} x'(a) > \sum_{a\epsilon E} y'(a)$$

which contradicts (2.10) and (2.12). Q.E.D.

Note that for economies where A is finite, core allocations may exist which cannot be achieved by exchanging goods with respect to a set of ℓ-1 relative prices.[2]

E. Aggregate exchange technologies

Because each individual executes his own exchanges, the aggregate exchange technology for a coalition E has the dimension of three times the cardinality of E. It is given by $\underset{a\epsilon E}{X} B(a)$ where X indicates ~~cross~~ Cartesian product.

Below we want to compare aggregate exchange technologies without division of labour with those with division of labour For this reason we represent the former in the space $R_+^{2\ell}$. For all $E\epsilon\ \Omega$, define the set AB(E) by

$$AB(E) = \{(u,v) \mid u,v\epsilon R_+^{\ell} \text{ and such that an allocation c exists attainable by E, with } u = \sum_{a\epsilon E} [c(a)-w(a)]^+,\ v = \sum_{a\epsilon E} [c(a)-w(a)]^- \}.$$

Note that

$$[c(a)-w(a)]_i^+ = \begin{cases} (c(a)-w(a))_i & \text{if } (c(a)-w(a))_i > 0 \\ 0 & \text{otherwise,} \end{cases}$$

$$[c(a)-w(a)]_i^- = \begin{cases} -(c(a)-w(a))_i & \text{if } (c(a)-w(a))_i < 0 \\ 0 & \text{otherwise,} \end{cases}$$

and that

$$[c(a)-w(a)]^+ = x(a)-z_2(a),$$
$$[c(a)-w(a)]^- = y(a)+z_1(a).$$

3. Exchange with Division of Labour

A. Traders

In the economy without division of labour in exchange an individual's exchange activities are tied to his preferences as a consumer. With division of labour an individual can act as a trader by buying goods and attempting to sell them at a profit. His trading activities are motivated by profit maximization, not consumption. Of course the profits he obtains as a trader will affect his budget set. However, we can separate, at least conceptually, his behaviour as a trader from his decisions as a consumer.

Express individual a's efficiency as a trader by his trading set T(a), a subset of $R_+^{2\ell}$. We say it is technologically feasible for trader a to purchase the vector v(a) from his customers and to sell the vector u(a)[4] if

$$(u(a),\ v(a)) \in T(a) \qquad (3.1)$$

The vector v(a)-u(a) represents the resources used up in the process. Material balance requires that v(a)-u(a)≥0. This requirement can be imposed on each trader's trading set T(a).

We assume that the following properties hold for every a∈A:

(T.1) 0 ∈T(a).

(T.2) If $(u(a),v(a))\epsilon T(a)$ and $0 \leq u'(a) \leq u(a)$ then $(u'(a), v(a))\epsilon T(a)$.

(T.3) T(a) is a closed, bounded (i.e. compact) set.

(T.4) T(a) is a convex set.[3]

Property (T.1) allows a trader to be inactive, property (T.2) permits free disposal of resources, property (T.3) ensures that a trader's scale of operations is finite. Property (T.4) implies non-increasing marginal returns.

B. The price system.

A trader's objective is to maximize profits. To cover transaction costs, a differential must exist between the prices his customers pay when they buy goods and receive when they sell goods to the trader. Denote this price vector by $(P_b, P_s)\epsilon R_+^{2\ell}$. Trader a attempts to engage in a set of trades $(\overline{u}(a),\overline{v}(a))\epsilon T(a)$ such that

$$P_b.\overline{u}(a)-P_s.\overline{v}(a)= \max\{P_b.u(a)-P_s.v(a)\,|\,(u(a),v(a))\epsilon T(a)\} \equiv \pi(a,P_b,P_s). \tag{3.2}$$

Thus $\pi(a,P_b,P_s)$ is trader a's maximum profit with respect to prices (P_b,P_s). Property (T.3) ensures that $(\overline{u}(a),\overline{v}(a))$ exists for each trader. Property (T.1) along with $(P_b,P_s)\geq 0$ imply that $\pi(a,P_b,P_s)\geq 0$.

In diagram 3.1 the maximum value of $P_b.u(a)-P_s.v(a)$ for $(u(a),v(a))\epsilon T(a)$ is obtained at $(\overline{u}(a),\overline{v}(a))$.

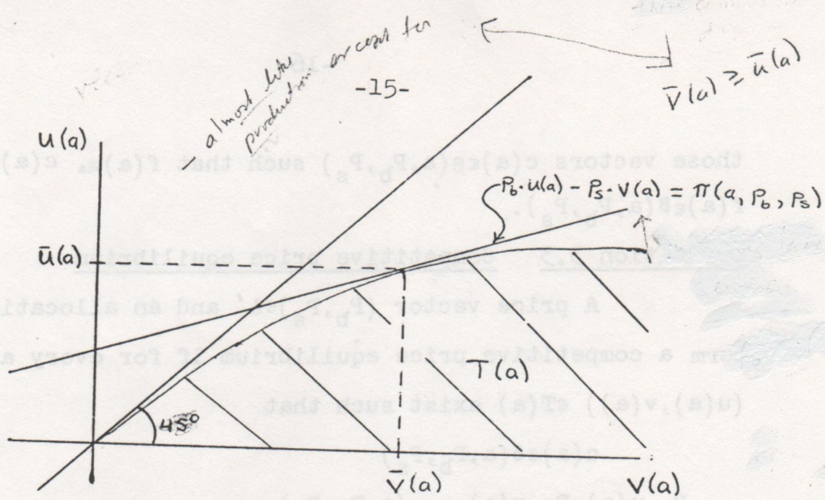

Diagram 3.1

As a trader an individual attempts to maximize profits; as a consumer he attempts to obtain the most desirable bundle of goods within his budget set. Define the unit price simplex $\Delta' \subset R_+^{2\ell}$ by

$$\Delta' = \{(P_b, P_s) \in R_+^{2\ell} \mid \sum_{i=1}^{\ell} (P_b^i + P_s^i) = 1\}.$$

Definition 3.1 Budget set.

Individual a's budget set $\beta(a, P_b, P_s)$ with respect to $(P_b, P_s) \in \Delta'$ consists of those vectors $c(a) \in X(a)$ such that

$$P_b[c(a)-w(a)]^+ \leq P_s \cdot [c(a)-w(a)]^- + \pi(a, P_b, P_s), \qquad (3.3)$$

where $[c(a)-w(a)]^+$ and $[c(a)-w(a)]^-$ are defined above. Note that $[c(a)-w(a)]^+$ and $[c(a)-w(a)]^-$ are, respectively, the goods received by a and the goods given up by a to obtain c(a) from w(a).

Definition 3.2 Demand set

Individual a's demand set $\delta(a, P_b, P_s)$ consists of those vectors $c(a) \epsilon \beta(a,P_b,P_s)$ such that $f(a) \precsim_a c(a)$ for all $f(a) \epsilon \beta(a,P_b,P_s)$.

Definition 3.3 Competitive price equilibrium

A price vector $(P_b,P_s) \epsilon \Delta'$ and an allocation c form a competitive price equilibrium if for every $a \epsilon A$ $(u(a),v(a)) \epsilon T(a)$ exist such that

$$c(a) \epsilon \delta(a,P_b,P_s) \tag{3.4}$$

$$P_b.u(a)-P_s.v(a) = \pi(a,P_b,P_s), \tag{3.5}$$

$$\sum_{a\epsilon A} [c(a)-w(a)]^+ = \sum_{a\epsilon A} u(a), \text{ and} \tag{3.6}$$

$$\sum_{a\epsilon A} [c(a)-w(a)]^- = \sum_{a\epsilon A} v(a). \tag{3.7}$$

In other words, c(a) is maximal with respect to $\precsim_a$ in each individual's budget set and the allocation C is attainable by profit maximizing trades (relations (3.5), (3.6), and (3.7)).

See [18] for the conditions under which a competitive price equilibrium exists.

C. Aggregate trade technologies and the core.

The trader's activities as described above were coordinated by a (P_b,P_s) price system. Let us assume that coalitions form not only for exchanging goods but also for co-ordinating trade. If a coalition assigns trading activities to its members, then its aggregate trade technology can be given by the set

$$AT(E)=\{(u,v)\epsilon R_+^{2\ell} \mid (u(a),v(a))\epsilon T(a) \text{ exist for every } a\epsilon E \text{ with } u = \sum_{a\epsilon E} u(a);\ v = \sum_{a\epsilon E} v(a)\}. \tag{3.8}$$

If $(u,v) \varepsilon AT(E)$ then coalition E can obtain the vector of goods v from its members and deliver the vector u to its members by a suitable assignment of trading activities to its members. Note that

$$AT(E) = \sum_{a \varepsilon E}' T(a) \qquad (3.9)$$

where Σ' indicates set theoretic sum. Given properties (T.1)-(T.4), then AT(E) satisfies the same properties. If E_1 and E_2 are coalitions, then

$$AT(E_1 \cup E_2) = AT(E_1) + AT(E_2). \qquad (3.10)$$

Aggregate trade technologies are additive over coalitions.

<u>Definition 3.4</u> <u>Attainable allocation (trade)</u>

An allocation c is attainable by the coalition E through trade if $(u,v) \varepsilon AT(E)$ exist such that

$$\sum_{a \varepsilon E} [c(a)-w(a)]^+ = u, \qquad (3.11)$$

$$\sum_{a \varepsilon E} [c(a)-w(a)]^- = v. \qquad (3.12)$$

<u>Definition 3.5</u> <u>Core allocation</u>

An allocation c which is attainable by the coalition A through trade is said to be a core allocation if it cannot be blocked by any coalition through trade.

D. <u>Individual and coalition traders</u>

A coalition E can be considered as a profit maximizing trader with technology AT(E). As a trader, a coalition buys goods from its members at one set of prices and resells them at another set. The differential in prices reflects the transaction costs incurred by the coalition as a trader. If we denote these prices by $(P_b, P_s) \epsilon R_+^{2\ell}$, then coalition E's profit with respect to (P_b, P_s) is given by

$$\Pi(E, P_b, P_s) = \max \{P_b . u - P_s . v \mid (u,v) \epsilon AT(E)\} \qquad (3.13)$$

In the definition of AT(E) we assumed that coalition E assigns trading activities to its members. We now show that a coalition can organize its technology of exchange as effectively with a (P_b, P_s) price system.

Theorem 3.1

If the individual traders for some coalition E are maximizing profits with respect to $(P_b, P_s)\epsilon\Delta'$, then the resulting assignment of trading activities maximizes profits for E with respect to (P_b, P_s).

Proof. Let $(u(a), v(a)) \epsilon T(a)$ for each $a \epsilon E$ be the profit maximizing trades with respect to (P_b, P_s). We claim that $(\sum_{a \epsilon E} u(a), \sum_{a \epsilon E} v(a))$ will maximize profits for E. Otherwise $(u', v') \epsilon AT(E)$ exists such that

$$P_b . \sum_{a \epsilon E} u(a) - P_s . \sum_{a \epsilon E} v(a) < P_b . u' - P_s . v' \qquad (3.14)$$

By definition of AT(E), $(u'(a), v'(a))$ exist for all $a \epsilon E$ such that $u' = \sum_{a \epsilon E} u'(a)$; $v' = \sum_{a \epsilon E} v'(a)$. Substituting into 3.14 we get

$$\sum_{a \epsilon E} (P_b u(a) - P_s v(a)) < \sum_{a \epsilon E} (P_b . u'(a) - P_s . v'(a)). \qquad (3.14)$$

But (3.14) can only be true if for some $\bar{a} \epsilon E$

$$P_b . u(\bar{a}) - P_s . v(\bar{a}) < P_b . u'(\bar{a}) - P_s . v'(\bar{a})$$

contradicting the fact that $(u(\bar{a}), v(\bar{a}))$ is a set of profit maximizing trades for $\bar{a}$. Proof by contradiction. Q.E.D.

Theorem 3.2.

If the coalition E as a trader is maximizing profits with respect to $(P_b, P_s) \epsilon \Delta'$, then the resulting assignment of trading activities maximizes profits for each individual trader with respect to (P_b, P_s).

Proof. Let $(u,v) \epsilon AT(E)$ such that

$$P_b.u - P_s.v = \max \{P_b.u' - P_s.v' \mid (u',v') \epsilon AT(E)\}.$$

By definition of AT(E), $(u(a), v(a)) \epsilon T(a)$ exist for each $a \epsilon E$ such that $u = \sum_{a \epsilon E} u(a)$; $v = \sum_{a \epsilon E} v(a)$. Then

$$P_b.u - P_s.v = \sum_{a \epsilon E} (P_b.u(a) - P_s.v(a)).$$

Suppose $\bar{a} \epsilon E$ exists with $(u'(\bar{a}), v'(\bar{a})) \epsilon T(\bar{a})$ such that

$$P_b.u(\bar{a}) - P_s.v(\bar{a}) < P_b.u'(\bar{a}) - P_s.v'(\bar{a}).$$

But this implies $(u',v') \epsilon AT(E)$ exists with

$$P_b.u - P_s.v < P_b.u' - P_s.v'$$

contradicting the selection of (u,v). Q.E.D.

Definition 3.6 Efficient trades

A set of trades $(u,v) \epsilon AT(E)$ is said to be efficient for coalition E if no $u' \epsilon R^{\ell}_{+}$ exists such that $u' > u$ and $(u',v) \epsilon AT(E)$.

Note that an efficient point is a boundary point of its respective trading set.

We state the following lemmas without proof.

Lemma 3.1

The profit maximizing set of trades with respect to prices (P_b, P_s) is an efficient point of AT(E).

Lemma 3.2

If $(u,v) \varepsilon AT(E)$ is efficient, then a price vector $(P_b, P_s) \geq 0$ exists such that (u,v) is a profit maximizing set of trades for coalition E.

Lemma 3.3

If $(u,v) \varepsilon AT(A)$ is the set of trades used to attain a core allocation, then (u,v) is efficient for the coalition A.

Theorem 3.3.

Every competitive equilibrium allocation is also a core allocation.[5]

Proof. Similar to that of Theorem 2.1.

4. The Choice of a Technology of Exchange

We now consider the choice between exchange without division of labour and trade. Generally one would expect that the latter would be more efficient then the former. However, this need not be true. For example, the (P_b, P_s) price system is more complicated then the p price system. More resources may be required to determine equilibrium prices for the (P_b, P_s) system and to disseminate them to individuals.

We contend that the use of a certain method of exchanging goods should result from individual and group maximizing behaviour. An individual who refuses to use the method that the rest of the economy is using will have no trading partners. Either he uses the economy's technology of exchange or else he must consume his initial endowment. To provide an alternative, we must consider the possibility that a group of individuals will break away from the economy and use another method to reallocate goods within the group.

The proof to the following theorem is trivial.

Theorem 4.1

If $AB(E) \subset AT(E)$ for each feasible coalition E, then trade will dominate exchange without trade.[6]

Proof. Let c be a core allocation using trade. Since $AB(E) \subset AT(E)$ for all E, an allocation which is attainable without division of labour is also attainable through trade. Therefore, c cannot be blocked by any coalition using exchange without division of labour.

Footnotes

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1. We assume that for every $p \epsilon \Delta$, each individual has sufficient wealth to exchange some positive quality of goods and still remain in the interior of his consumption set. See [18] for the conditions when this is true.

2. In [18] we show that if A has the cardinality of the continuum then every core allocation is also a competiti equilibrium allocation for some $p \epsilon \Delta$.

3. The set T(a) can be obtained from B(a) as follows. $T(a)=\{(x(a)-z_1(a), y(a)+z_2(a)) \mid (x(a),y(a),z(a)=z_1(a)+z_2(a))\epsilon B(a)\}$ If (B.1)-(B.4) hold then (T.1)-(T.4) hold.

4. We assume that transaction costs depend on the volume of trade, not on the identity and number of customers.

5. In [18] we show that if A has the cardinality of the continuum, then every core allocation using trade is also a competitive equilibrium allocation for some $(P_b,P_s)\epsilon\Delta'$.

6. The relation $AB(E)\subset AT(E)$ for each E holds if T(a) is obtained from B(a) as described in footnote 3.

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